A Two-Axis Phase Diagram for Market-Regime Classification

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Abstract

Existing approaches to market-regime classification typically use a discrete state space indexed by a hidden Markov model (Hamilton, 1989) over scalar observables (VIX, yield curve, realised volatility). We propose an alternative geometric classification in the two-dimensional plane spanned by (i) the instanton action S_{inst} on a bistable correlation potential (Coleman, 1977; Andreev & Howden, 2026b) and (ii) the spontaneous-symmetry-breaking condensate magnitude $|\Phi|^2$—the dominant principal-eigenvalue weight in the correlation matrix. The two axes are mathematically independent and have direct physical interpretations: S_{inst} measures the distance from a correlation-state phase transition; $|\Phi|^2$ measures the degree of factor-structure alignment. The (S_{inst}, $|\Phi|^2$) plane partitions naturally into four quadrants: high-action high-condensate (healthy trend); high-action low-condensate (diversified calm); low-action high-condensate (crowded stress); low-action low-condensate (pre-transition chop). For each quadrant we describe the correspondence to a strategy family at the mathematical-framework level (without specifying proprietary strategy implementations). A proposed (not yet registered) prospective validation protocol is described — the programme's actually-registered prospective object for the phase diagram is the narrower phase-quadrant crisis-filter hypothesis, frozen 2026-04-18 (§5); **a prior internal claim of "2–4 day lead time over HMM" is formally retracted**—the contribution of this paper is the classification scheme itself, not any specific empirical lead-time result. The one retrospective result we do report is negative: a naive quadrant-driven risk-on/off allocation underperforms a fixed 60/40 benchmark out-of-sample (2015–2024, net of costs), which is why no allocation-alpha claim accompanies the classification.

1. Introduction

Quantitative finance has accumulated multiple distinct regime-classification methods. Hidden Markov Models classify regimes as discrete states over scalar volatility and macro observables (Hamilton, 1989; Ang & Bekaert, 2002). Bayesian factor-timing approaches (AQR; Asness *et al.*, 2013) blend continuous evidence into regime posteriors. Topological methods compute persistent homology of correlation networks (Gidea & Katz, 2018). Each method commits to a specific functional form for the regime indicator; none directly use the underlying physical order parameters of the correlation dynamics.

The stochastic-mechanical framework (Andreev & Howden, 2026a) provides a natural pair of order parameters for market dynamics:

Two physical axes for market regimes.
- S_{inst} — the Euclidean (Coleman bounce) action separating the normal-correlation basin from the crisis-correlation basin on a bistable potential. Large S_{inst} means the barrier between basins is high; transitions are rare. Small S_{inst} means the barrier is shallow; transitions are likely.
- $|\Phi|^2$ — the SSB condensate magnitude, equal to the mean-field projection of the inverse correlation matrix onto its dominant eigenvector. Large $|\Phi|^2$ means a single factor dominates (broken symmetry, e.g., during a strong bull or bear trend). Small $|\Phi|^2$ means assets behave near-independently (symmetric phase).

These two axes are mathematically independent: a market can have low barrier action (S_{inst} small) without strong factor alignment, and vice versa. The joint (S_{inst}, $|\Phi|^2$) plane partitions into four quadrants with distinct physical meaning.

> **Contribution.** We propose a four-quadrant phase diagram in the (S_{inst}, $|\Phi|^2$) plane as a market-regime classification scheme. For each quadrant we describe the strategy-family correspondence at the mathematical-framework level. We propose a prospective validation protocol — not yet registered; the register's frozen prospective object is the narrower phase-quadrant crisis-filter hypothesis (§5) — for testing whether quadrant transitions lead, lag, or coincide with classical HMM regime transitions. We formally retract an earlier internal claim of "2–4 day lead over HMM" based on insufficient out-of-sample evidence; the contribution of this paper is the classification scheme, not the lead-time number.

2. The Two Order Parameters

2.1 Instanton action S_{inst}

Following Andreev & Howden (2026b), the correlation order parameter $\rho(t)$ is the mean-field projection of the asset-asset correlation matrix $\Sigma(t)$. It evolves under a bistable double-well potential:

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V(ρ) = (λ/4) (ρ - ρₙ)² (ρ - ρ꜀)²
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with ρ_n the normal-correlation basin centre and ρ_c the crisis-correlation basin centre. The Euclidean (instanton) action separating the two basins is computed from the classical Euclidean trajectory under the inverted potential $-V(\rho)$:

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Sᵢₙₛₜ(t) = ∫_ρₙ^ρ꜀ √(2 V(ρ)) dρ
                  = (2√2 / 3) · √λ(t) · |ρ꜀(t) -
ρₙ(t)|³
```

Large S_{inst} indicates a deep, sharp barrier between the two basins: the system is stably in one basin (typically the normal one) and transitions are rare. Small S_{inst} indicates a shallow barrier: the system can transition with low energy cost. The instanton action is calibrated either from option-implied densities (Andreev & Howden, 2026g) or from rolling-window correlation history (this paper).

2.2 SSB condensate $|\Phi|^2$

The SSB condensate magnitude is the mean-field projection of the inverse correlation matrix $\Sigma(t)^{-1}$:

```
|Φ|²(t) = (1/N²) · Σᵢ,ⱼ Σ⁻¹ᵢⱼ(t)
                   normalised to [0, 1] across
the sample
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An equivalent computation in terms of eigenvalues: $|\Phi|^2(t) = \lambda_1(t)/\Sigma_k\lambda_k(t)$ — the fraction of total correlation variance explained by the dominant principal direction (the market-mode eigenvalue of random-matrix analyses of financial correlations; Plerou *et al.*, 2002). Large $|\Phi|^2$ means a single factor dominates (consensus regime, e.g., strong trend); small $|\Phi|^2$ means eigenvalues are nearly degenerate (diversified regime).

2.3 Independence of the axes

The two order parameters are mathematically distinct functionals of the correlation history. The instanton action depends on the shape of the bistable potential and its barrier; the condensate magnitude depends on the eigenvalue distribution of the correlation matrix. Their empirical dependence must be measured, not asserted. The one calibration run archived to date (the 2015–2024 daily classification of Section 5.1; 2,516 trading days on a five-ETF observation universe) offers indirect evidence: all four quadrants are materially populated (occupancy 10.9–41.1% of days; Figure 1), and the two axis-aligned quadrants jointly hold 48.0% of days, against the 45.4% that statistical independence of the dichotomised axes at their observed marginal frequencies would produce—a mild positive dependence, far from redundancy. Were the axes strongly redundant, occupancy would concentrate in the two aligned quadrants. A formal rank-correlation report between the raw axis series is part of the prospective protocol of Section 5.

3. The Four Quadrants

The (S_{inst}, $|\Phi|^2$) plane partitions naturally into four quadrants by thresholding each axis at its sample median (or empirically meaningful percentile). Each quadrant has a distinct physical and strategic interpretation.

Table 1: Four-quadrant phase diagram

QUADRANT	S_{INST}	$|\Phi|^2$	INTERPRETATION	STRATEGY CORRESPONDENCE
I — Healthy Trend	High	High	Strong factor alignment, far from transition barrier	Trend-following / momentum methods family
II — Diversified Calm	High	Low	Independent assets, far from transition	Mean-variance / risk-parity methods family
III — Crowded Stress	Low	High	Strong factor alignment, close to transition barrier	Hedging / volatility methods family
IV — Pre-Transition Chop	Low	Low	Diversified but near a barrier; regime change incipient	Tail-risk / instanton-aware methods family

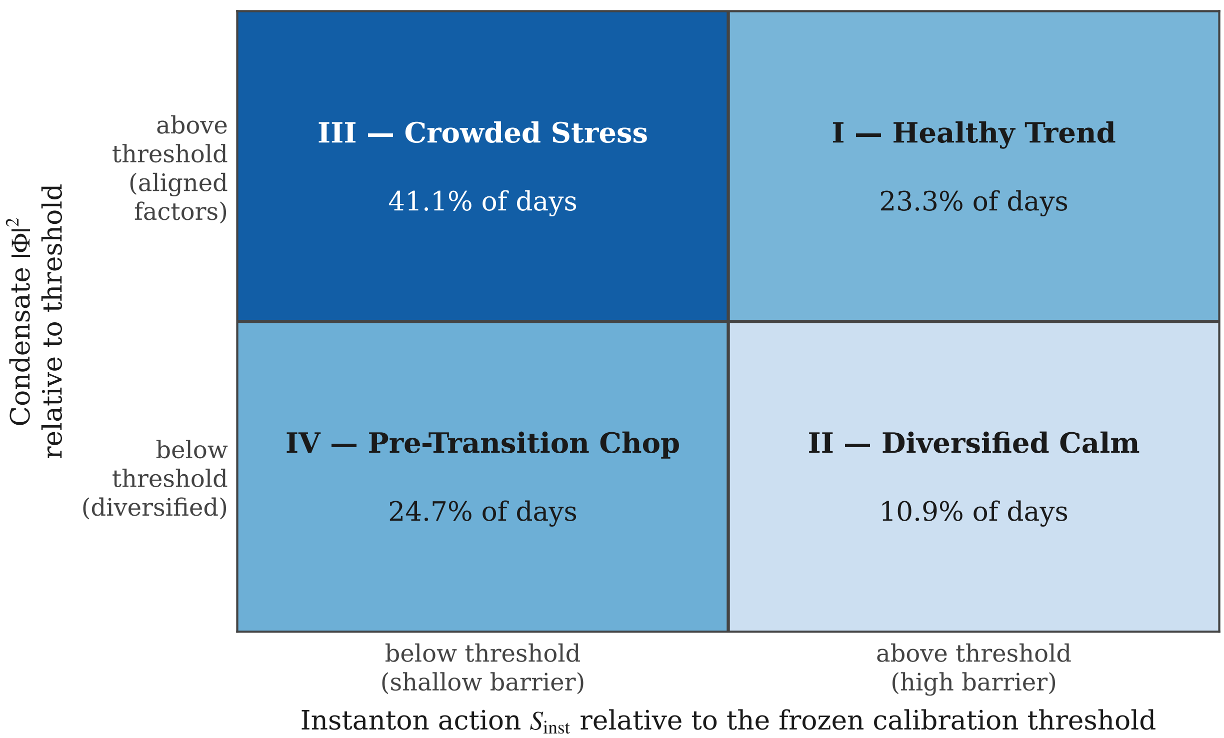

Figure 1. Quadrant occupancy in the (S_{inst}, $|\Phi|^2$) plane over the archived 2015–2024 daily classification run (2,516 trading days; both order parameters observed on a five-ETF universe of SPY, QQQ, IWM, TLT and GLD; darker shading = larger share of days). Quadrant boundaries are the frozen calibration thresholds fitted on the first three years of the sample (S_{inst} threshold 0.218, $|\Phi|^2$ threshold 0.242). All four quadrants are materially populated, and the shallow-barrier, factor-aligned Crowded Stress quadrant is the modal state (41.1% of days)—the plane does not collapse onto either axis. Quadrant labels follow Table 1; the live classifier's internal names map I = stable growth, II = fragmented calm, III = structured stress, IV = crisis. Data: the archived phase-diagram backtest results file (April 2026).

3.1 Quadrant I — Healthy Trend

High S_{inst} and high $|\Phi|^2$ together describe a regime in which a single factor (typically broad equity beta or commodity beta) dominates the correlation structure, AND the barrier between normal and crisis correlation states is well above the typical fluctuation scale. The market is in a stable consensus direction. Strategy correspondence: methods that ride the dominant factor.

3.2 Quadrant II — Diversified Calm

High S_{inst} and low $|\Phi|^2$ describes a regime in which assets behave near-independently (no single factor dominates) AND the barrier between basins is high (no incipient transition). This is the textbook environment for mean-variance and risk-parity portfolio construction: the cross-section is well-conditioned, correlations are moderate, and the system is far from any phase transition.

3.3 Quadrant III — Crowded Stress

Low S_{inst} and high $|\Phi|^2$ describes a regime in which a single factor dominates the cross-section AND the barrier to crisis-correlation is shallow. Crowded positioning is fragile: any shock can trigger correlated unwinding. This is the classic late-stage bull or late-stage bear regime preceding a sharp regime change. Strategy correspondence: hedging methods and tactical de-risking; the crowded-trade reading of the osmotic-velocity scaling result (Andreev & Howden, 2026h) applies here.

3.4 Quadrant IV — Pre-Transition Chop

Low S_{inst} and low $|\Phi|^2$ describes a regime in which assets are individually noisy (no consensus direction) AND the barrier between basins is shallow. The system is poised for a regime change but has not yet committed to a direction. Empirically this quadrant tends to occur in transitional periods between trending regimes (e.g., late-stage bull turning into early-stage bear). Strategy correspondence: tail-risk methods, including the instanton-aware combined-VaR estimator (Andreev & Howden, 2026g) and the combined TDA×instanton crash signal (Andreev & Howden, 2026i).

4. Comparison with Existing Classification Schemes

4.1 Hidden Markov Models

HMM-based regime detection (Hamilton, 1989; Ang & Bekaert, 2002) assigns each observation to one of a small number of discrete latent states based on Gaussian emissions for scalar observables. The classification is committed to a fixed number of states (typically 2–4) and to a forward–backward inference procedure for state assignment.

The phase-diagram classification differs in three ways. (i) The classification axes are continuous and physically interpretable; the quadrant assignment emerges from thresholding, not from a hidden-state inference. (ii) Transitions between quadrants reflect motion in the underlying potential and condensate, which is dynamically structured by the bistable correlation dynamics. (iii) The classification scales to higher resolutions (e.g., octants via three axes; cf. Andreev & Howden, 2026d for the factor-stability third axis) without changing the underlying mathematical framework.

4.2 Bayesian factor timing

Bayesian factor-timing approaches combine evidence from multiple scalar observables into a regime posterior (Asness *et al.*, 2013). The phase-diagram classification is complementary: each of S_{inst} and $|\Phi|^2$ is a single observable that can feed into a Bayesian posterior alongside others. We do not claim that the phase-diagram classification subsumes Bayesian methods; we claim that the two axes are physically motivated additions to the observable set.

4.3 Topological methods

Persistent homology methods (Gidea & Katz, 2018) detect topology change in the correlation network. The relationship to the phase-diagram axes: large persistent H_1 features tend to coincide with high $|\Phi|^2$

(topology-detected loops correspond to factor-alignment regimes); the instanton action is a separate channel reflecting potential-shape rather than network topology. The combined TDA×instanton crash signal (Andreev & Howden, 2026i) is a special case of using both kinds of channel; we note that its own empirical verdict is threshold-dependent and ambiguous (a tight-threshold configuration passes and an aggressive-threshold configuration fails the Harvey bar), so it is cited here as a methodological relative, not as a validated predictor.

5. Proposed Validation Protocol

> **Retraction of prior claim.** An earlier internal analysis claimed "2–4 day lead time over HMM" for the phase-diagram classification. The claim was based on retrospective analysis of curated historical windows and is *not* supported by held-out forward evidence under Harvey–Liu–Zhu (2016). We formally retract that result here. The lead-time question is open and is the subject of the proposed prospective validation protocol described below.

> **Proposed prospective protocol (not yet registered).**
> **(P1)** Universe: 30 most-liquid US-traded ETFs (frozen ex-ante).
> **(P2)** Computation: $S_{\text{inst}}(t)$ and $|\Phi|^2(t)$ computed daily on rolling 60-day correlation matrix.
> **(P3)** Thresholding: quadrant assignment by the sample median of each axis, computed on a 2010–2017 in-sample window; thresholds frozen.
> **(P4)** Baseline: 4-state HMM on VIX, term structure slope, realised vol of SPY, yield-curve slope.
> **(P5)** Outcome metric: lead-time correlation between phase-diagram quadrant transitions and HMM regime transitions, computed on rolling 100-day windows in the 2018–present out-of-sample period.
> **(P6)** Significance: cross-correlation lag at peak $|\rho|$; significance via block-bootstrap with block length 20 days, $B = 1000$. Harvey $t > 3.0$ required to claim either lead or lag relative to HMM.
> **(P7)** Status: empirical lead-time results will be reported as a separate sequel paper. We make no Harvey-significance claim in the present paper; the only retrospective result reported here is the negative allocation test of Section 5.1.

For clarity of registration status: the P1–P7 lead-time-versus-HMM protocol above is a *proposal* — it does not yet exist in the research programme's hypothesis register, and running it will require its own fresh registration. The register's actually-frozen prospective object for the phase diagram is the narrower *phase-quadrant crisis-filter* hypothesis (frozen 2026-04-18): quadrant thresholds frozen at $S_{\text{inst}} = 0.218$ and $|\Phi|^2 = 0.242$, the outcome measured on forward 20-day SPY windows, the verdict deferred until at least five prospectively-observed crisis windows have accumulated under those frozen thresholds, and the classifier's live plumbing already wired for that accrual. The two protocols are complementary: the registered crisis filter tests the quadrant scheme's crisis discrimination prospectively, while the proposed P1–P7 sequel would test the broader lead-time question.

5.1 A retrospective allocation test (negative result)

Independently of the open lead-time question, one retrospective test of the quadrant scheme has been run and archived, and we report it here because it is negative and therefore bounds what the classification can be claimed to deliver. The test drives a daily risk-on/off weight on a two-asset (SPY, TLT) book from the quadrant classifier's regime score, with S_{inst} and $|\Phi|^2$ observed on a five-ETF universe (SPY, QQQ, IWM, TLT, GLD) over 2015–2024 (2,516 trading days). Thresholds were calibrated on the first three years and frozen; the remaining ~8 years are out-of-sample. Rebalancing is smoothed (5-day EMA) and charged 10 basis points per unit of turnover.

Table 2: Retrospective quadrant-allocation test, 2015–2024, net of 10 bps costs

BOOK	SHARPE	ANN. RETURN	ANN. VOL	MAX DRAWDOWN	CALMAR	TOTAL RETURN
Quadrant-driven allocation	0.56	7.8%	13.8%	−29.6%	0.26	+97.4%
Fixed 60/40	0.75	8.3%	11.1%	−27.2%	0.30	+114.4%
Buy-and-hold SPY	0.78	13.8%	17.6%	−33.7%	0.41	+239.4%

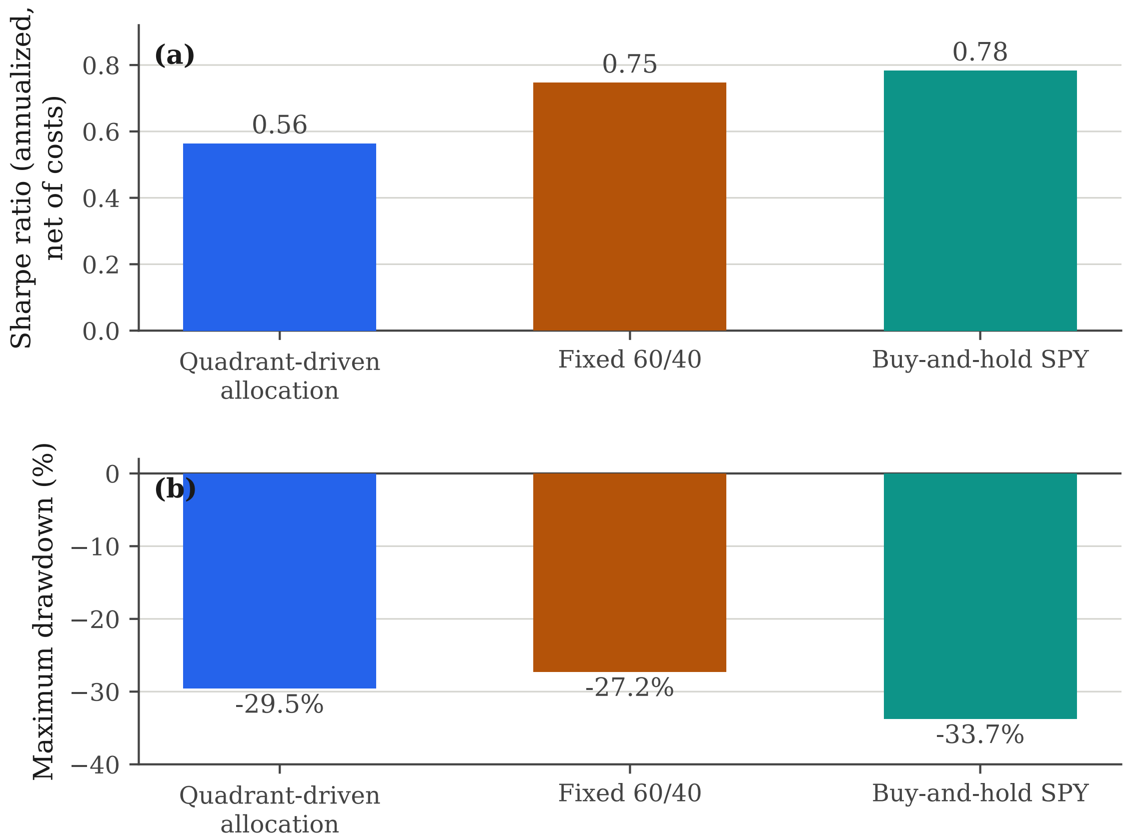

Figure 2. The retrospective allocation test behind the paper's no-alpha stance. Panel (a): annualized Sharpe ratio, net of 10 bps transaction costs, of the naive quadrant-driven risk-on/off allocation on (SPY, TLT) versus a fixed 60/40 book and buy-and-hold SPY, 2015–2024 with thresholds frozen on the first three years. Panel (b): maximum drawdown over the same window. The quadrant-driven book underperforms the static 60/40 on Sharpe (0.56 vs 0.75) without buying a better drawdown (−29.6% vs −27.2%): the classification, applied naively as an allocation signal, subtracts value after costs. Data: the archived phase-diagram backtest results file (April 2026).

What the negative result does and does not say. It says that thresholded quadrant membership, mapped through a fixed per-quadrant risk weight and rebalanced with a smoothing filter, does not beat a static 60/40 on this universe and period after costs (archived verdict: the phase book underperforms the 60/40 benchmark). It does not test the lead-time question of the prospective protocol (P1–P7), and it does not test richer uses of the axes (e.g., as two observables inside a larger posterior, Section 4.2). But it is the reason this paper claims a classification scheme and not an allocation edge: the one allocation-style test run to date lost to its benchmark.

6. Relation to Other Quark Papers

The phase-diagram classification ties together several other contributions in the Quark research series. The instanton-action axis is calibrated using the RG-flow construction of Andreev & Howden (2026b) and the non-perturbative VaR construction of (2026g). The condensate-magnitude axis is the same quantity used in the osmotic-velocity scaling paper (2026h) and the density-matrix ensemble allocator of the main Pilot-Wave Finance whitepaper. The strategy correspondences per quadrant link to the Stochastic Methods paper (2026a) and the combined TDA×instanton crash signal (2026i).

A natural extension is the three-axis "phase cube," in which the factor-stability index (Andreev & Howden, 2026d) is added as a third independent axis. The phase cube partitions into eight octants with finer-resolution regime correspondence; a separate paper on this extension is in preparation.

7. Discussion

The four-quadrant phase diagram is a regime-classification framework, not an empirical claim about lead times or predictive accuracy. The classification is well-motivated from the underlying physics: S_{inst} and $|\Phi|^2$ are physically distinct order parameters of the correlation dynamics, and their joint state captures information that cannot be recovered from a scalar HMM observable. Whether this additional information translates into forward-predictive value over HMM is the subject of the proposed validation protocol in §5, with the registered phase-quadrant crisis-filter hypothesis providing the currently-frozen prospective test. The retrospective evidence available so far cuts the other way: the one archived allocation-style test of the quadrant scheme (§5.1) underperformed a static 60/40 after costs, so any eventual value must come from information content (the lead-time question), not from the naive threshold-to-weight mapping already tested.

The retraction of the earlier "2–4 day lead" claim is intentional and follows the methodological discipline of Andreev & Howden (2026e): retrospective tests on curated historical windows do not constitute Harvey-significance, and the publishable result is the classification scheme itself, the prospective protocol, and (eventually) the held-out out-of-sample test result.

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