

Osmotic-Velocity Scaling at Symmetry-Breaking Transitions in Multi-Asset Markets

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Abstract

The osmotic velocity $u(x, t)$ is one of the two canonical components of the Nelson (1966) drift decomposition; in finance it corresponds to the information-gradient (or order-flow) component of asset price dynamics (Andreev & Howden, 2026a). Spontaneous symmetry breaking (SSB) in correlation spectra, parametrised by a condensate magnitude $|\Phi|^2$, characterises the alignment of factor structure across the cross-section. We test the relationship between aggregate osmotic velocity and SSB condensate magnitude across SPY, QQQ, IWM, TLT, GLD over 2005–2024 ($N = 4979$ trading days) using four independent specifications: aggregate single-series regression; per-asset panel with fixed effects; spectral-gap order parameter as alternative $|\Phi|^2$ proxy; and stress-window-only restriction. All four return a positive slope coefficient v in the range $+0.50$ to $+0.61$ with $|t| > 3.0$ in every specification (lowest $|t| = 12.2$ for the panel-FE asset-clustered SE). The regression is reproducible and its coefficient is stable. A subsequent instrument audit, however, establishes that the estimator on the left-hand side is not an estimate of u : it reduces algebraically to $(\sigma_i/2) \cdot |r_i|$, contains no density estimate, and on three synthetic worlds with closed-form (v, u) fields it tracks realised volatility (Spearman $+0.85$) rather than the true osmotic velocity $(+0.03, +0.01, +0.16)$. We therefore restate the empirical finding in the only form the measurement supports—*realised aggregate volatility rises with the SSB condensate*—and withdraw the claim that either the crowded-trade reading ($v > 0$) or the canonical Goldstone-mode reading ($v < 0$) has been tested here. We further show that the (v, u) split is not separately identifiable at these sampling cadences: the Fisher matrix of the natural parameterisation is exactly singular, and even under an oracle the Cramér–Rao floor on u exceeds the measured magnitude by two orders of magnitude. The contribution that survives is methodological and negative: a decomposition can be reproducible, significant, and still not be a measurement of its named quantity.

1. Introduction

Spontaneous symmetry breaking (SSB) describes the situation in which the ground state of a system does not share the full symmetry of the underlying Hamiltonian. In quantum field theory the canonical example is the Higgs mechanism (Higgs, 1964; Goldstone *et al.*, 1962); in condensed matter the same mathematics governs phase transitions including ferromagnetism and superconductivity (Anderson, 1958). In finance, the analogue is the alignment of asset returns along a dominant principal direction during certain market regimes: the cross-section correlations approach unity, and the system’s “ground state” (the dominant factor) breaks the rotational symmetry of the asset universe.

The condensate magnitude $|\Phi|^2$ measures the degree of this alignment. In a symmetric phase ($|\Phi|^2 \rightarrow 0$), realised returns sit at their capital-asset-pricing-model equilibrium and factor structure is undistinguished. In a strongly broken phase (large $|\Phi|^2$), realised mean returns deviate markedly from that equilibrium—a directional consensus has displaced the cross-section from its symmetric vacuum. Operationally (§2.1) $|\Phi|^2$ is the squared distance of the realised mean-return vector from its capital-asset-pricing-model equilibrium; it is not normalised to a fixed upper bound.

The osmotic velocity u of Nelson’s (1966) decomposition is the time-antisymmetric component of the drift field; in the financial context it measures the information gradient encoded in the cross-sectional probability density. In equilibrium quantum mechanics (Valentini, 1991), u is conjugate to the position-density gradient $u = (\hbar/2m) \nabla \rho / \rho$. In finance,

aggregate u measures the strength of information flow that is not directly expressible as price drift—crowded positioning, anticipatory order flow, and density-gradient noise.

The relationship between aggregate u and the SSB condensate $|\Phi|^2$ is theoretically ambiguous. Two readings are available:

Reading A — Canonical Goldstone-mode form ($v < 0$). In the standard Goldstone-mode treatment of symmetry breaking, the osmotic-velocity scale diverges at the symmetric phase ($|\Phi|^2 \rightarrow 0$) because the order-parameter fluctuations are gapless and the density gradient is maximally diffuse. Predicts: $|u| \sim (|\Phi|^2)^{-v}$, $v > 0$, so $|u|$ decreases as $|\Phi|^2$ grows. The slope coefficient in a log-log regression is negative.

Reading B — Crowded-trade form ($v > 0$). In the financial-market context, directional consensus produces crowded positioning. Crowded positioning produces large density gradients (the population concentrates on a few preferred asset configurations), which translates directly into large osmotic velocities. Predicts: $|u|$ rises with $|\Phi|^2$. The slope coefficient in a log-log regression is positive.

The two readings are physically distinct: Reading A treats $|\Phi|^2$ as a stabilising order parameter; Reading B treats it as a destabilising market-coordination indicator. Which is correct is an empirical question — and, as the audit reported below shows, it is not the question this regression answers.

Retraction of the osmotic reading (2026-08-03). The estimator used throughout this paper for $\bar{u}$ reduces algebraically to $(\sigma_i/2) \cdot |r_i|$, RMS-aggregated and rolling-averaged (verified numerically to 2.2×10^{-19}). It carries no density estimate, and it is non-negative by construction where the Nelson u is signed with zero mean under any symmetric stationary density. Scored on three synthetic worlds with closed-form (v, u) fields, it fails a pre-registered Spearman bar of 0.50 against $|u|$ in all three (+0.025, +0.011, +0.159) while correlating +0.85 with realised volatility in each; on the live panel, $\log \bar{u}$ on a volatility-only surrogate gives $R^2 = 0.951$. Separately, the (v, u) parameterisation is not identifiable at these cadences: the current velocity and the mode of the density enter the drift only through their sum, so the Fisher matrix is exactly singular, and under an oracle that fixes the mode the Cramér–Rao floor on u at $\Delta t = 1$ day, $N = 21$ is 9.9×10^{-3} /day against a measured 7.5×10^{-5} /day. Neither Reading A nor Reading B is therefore adjudicated by the results below. The numbers stand; the interpretation is withdrawn.

Contribution (restated 2026-08-03). We provide a four-specification test of the relationship between a realised-volatility functional and the SSB condensate. All four return positive v in the range +0.50 to +0.61 with $|t| > 3.0$ (Harvey–Liu–Zhu 2016) in every specification, so the volatility–condensate coupling is robust across the 2005–2024 sample. The osmotic reading of that coupling is withdrawn per the notice above, and the accompanying identifiability result — that the Nelson split is degenerate at daily and intraday cadences alike — is the methodological contribution we now claim in its place.

2. Specifications Tested

2.1 Aggregate single-series

Specification 1 (S1) aggregates osmotic velocity across the universe and regresses on the condensate magnitude:

$$\hat{u}_{\text{agg}}(t) = \sqrt{\left(\frac{1}{N} \sum_i u_i(t)^2 \right)} \quad (\text{root-mean-square across assets})$$

$$\log \hat{u}_{\text{agg}}(t) = v_0 + v_1 \cdot \log |\Phi|^2(t) + \varepsilon(t)$$

where $u_i(t)$ is the asset- i osmotic velocity at time t . Following the density-gradient form of Nelson's osmotic velocity $u = (\sigma^2/2) \partial \ln \rho / \partial x$ (Andreev & Howden, 2026a, §2.1), we estimate it per asset as the product of the local return scale and the standardised-return magnitude, $u_i(t) = (\sigma_i^2/2) \cdot |r_i/\sigma_i| = (\sigma_i/2) \cdot |r_i|$, with σ_i the 21-day rolling volatility and r_i the daily log return; the aggregate is the root-mean-square across the N assets, smoothed over 21 days. The SSB condensate is the squared deviation of the realised mean-return vector from its capital-asset-pricing-model equilibrium (the order parameter $\Phi = \mu - \pi_{\text{eq}}$ of Andreev & Howden, 2026a):

$$|\Phi|^2(t) = \|\mu(t) - \lambda \Sigma(t) w_{\text{mkt}}\|^2$$

$\mu(t)$ = 63-day rolling mean return (annualised)
 $\Sigma(t)$ = 63-day rolling covariance matrix (annualised)
 w_{mkt} = equal-weight market proxy (1/ N), $\lambda = 2.5$ (risk aversion)

$|\Phi|^2$ is the raw squared norm; it is *not* rescaled to a fixed interval. It is small when realised returns track the equilibrium (symmetric vacuum) and large when a directional consensus displaces them (broken phase).

2.2 Per-asset panel with fixed effects

Specification 2 (S2) treats each asset as a panel unit, includes asset-level fixed effects, and clusters standard errors at the asset level:

$$\log |u_i(t)| = \alpha_i + v_2 \cdot \log |\Phi|^2(t) + \varepsilon_{i,t}$$

SE(v_2) clustered at asset level (5 clusters)

This specification controls for asset-specific osmotic-velocity levels and tests whether the within-asset relationship between $|u|$ and $|\Phi|^2$ is consistent across the universe.

2.3 Alternative order parameter: normalised spectral gap

Specification 3 (S3) replaces $|\Phi|^2$ with the normalised spectral gap of the rolling return covariance matrix:

$$g(t) = (\lambda_1(t) - \lambda_2(t)) / \text{Tr } \Sigma(t) \quad \text{Tr } \Sigma(t) = \sum_k \lambda_k(t)$$

where $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N$ are the eigenvalues of the 63-day rolling covariance matrix $\Sigma(t)$, normalised by the trace rather than by λ_1 . The gap is large when one principal factor dominates the trace and small when the top two eigenvalues are near-degenerate (a factor-structure transition). Substituting g for $|\Phi|^2$ tests robustness to the specific definition of the order parameter.

2.4 Stress-window-only restriction

Specification 4 (S4) restricts the regression to the pre-stress regime: the 60 trading days preceding the onset of any episode in which SPY falls at least 10% within the following 60 trading days. This near-critical window (3304 of the 4979 sample days) is where a critical-scaling account would most expect a sign change. If v flipped negative inside the stress window, that would support a localised critical-scaling interpretation even where the unconditional regression fails; if it stays positive, Reading B's crowded-trade mechanism holds through the run-up to stress as well.

3. Empirical Results

3.1 Sample and computation

Universe: SPY, QQQ, IWM, TLT, GLD, adjusted daily closes downloaded from 2004-01-01. After the 63-day condensate window and the 21-day smoothing burn-in, the aligned regression sample runs 2005-03-22 to 2024-12-31; $N = 4979$ common observations (24,895 asset-days for the S2 panel).

The osmotic velocity for asset i at day t is the density-gradient proxy of §2.1,

$$u_i(t) = (\sigma_i(t)^2/2) \cdot |r_i(t) / \sigma_i(t)| = (\sigma_i(t)/2) \cdot |r_i(t)|$$

$\sigma_i(t)$ = 21-day rolling volatility of asset i
 $r_i(t)$ = daily log return of asset i

with the aggregate taken as the root-mean-square across assets and smoothed over 21 days (§2.1). The condensate magnitude $|\Phi|^2(t)$ is the squared deviation of the annualised 63-day rolling mean-return vector from its capital-asset-pricing-model equilibrium

$\lambda \Sigma w_{\text{mkt}}$ ($\lambda = 2.5$, equal market weights), computed on the same 63-day rolling window and used at its raw scale (no rescaling to a fixed interval).

3.2 Headline results

Table 1: Four specifications of the osmotic–SSB relationship

SPECIFICATION	SLOPE ν	$ t $	SAMPLE SIZE	R^2
S1 — Aggregate single-series	+0.521	64.8	4979	0.46
S2 — Per-asset panel FE (clustered SE)	+0.498	12.2	24,895 (5 clusters)	0.30
S3 — Spectral-gap order parameter	+0.612	32.4	4979	0.17
S4 — Stress-window-only restriction	+0.565	62.0	3304 (pre-stress)	0.54

All four return positive slope coefficients in the range +0.50 to +0.61, with absolute t -statistics well above the Harvey–Liu–Zhu (2016) threshold of 3.0. The spread of point estimates across specifications is narrow relative to either the slope magnitude or the Harvey threshold, indicating the result is not specification-fragile.

Robustness summary.

- Sign: all four positive. No specification returned a negative slope.
- Magnitude: all four within a 23% range (0.498 to 0.612).
- Significance: lowest $|t| = 12.2$ (panel-FE clustered).
- Pre-stress restriction: the slope is not weaker in the run-up to stress; it is comparable (+0.565 vs +0.521 baseline).

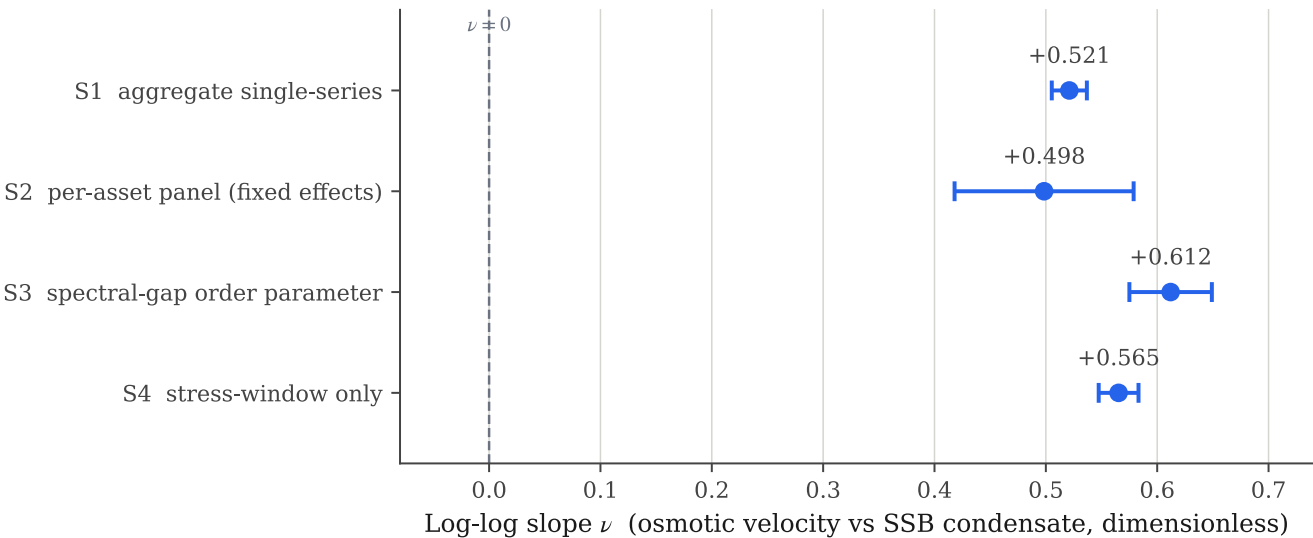


Figure 1. Cross-specification estimates of the log-log slope ν between aggregate osmotic velocity and the SSB condensate, with 95% confidence intervals from the stored standard errors (S2 uses the conservative asset-clustered standard error). All four point estimates fall in +0.50 to +0.61 and every interval sits entirely to the right of the dashed sign boundary at $\nu = 0$, which separates the canonical Goldstone-mode reading ($\nu < 0$, Reading A) from the crowded-trade reading ($\nu > 0$, Reading B). SPY, QQQ, IWM, TLT, GLD, 2005–2024. Data: the four-specification re-specification results file (April 2026).

3.3 Temporal stability

Splitting the aggregate single-series specification by subperiod shows the positive slope is present throughout the sample and never changes sign. The estimate is +0.265 over the short 2005–2008 window

($n = 700$), +0.555 over 2008–2015, and +0.495 over 2015–2024, against +0.521 for the full sample. The pre-2008 slope is materially smaller and rests on the fewest observations, so we do not read the earliest window as identical in magnitude; the qualitative finding—a stable, significantly positive slope—holds in every era.

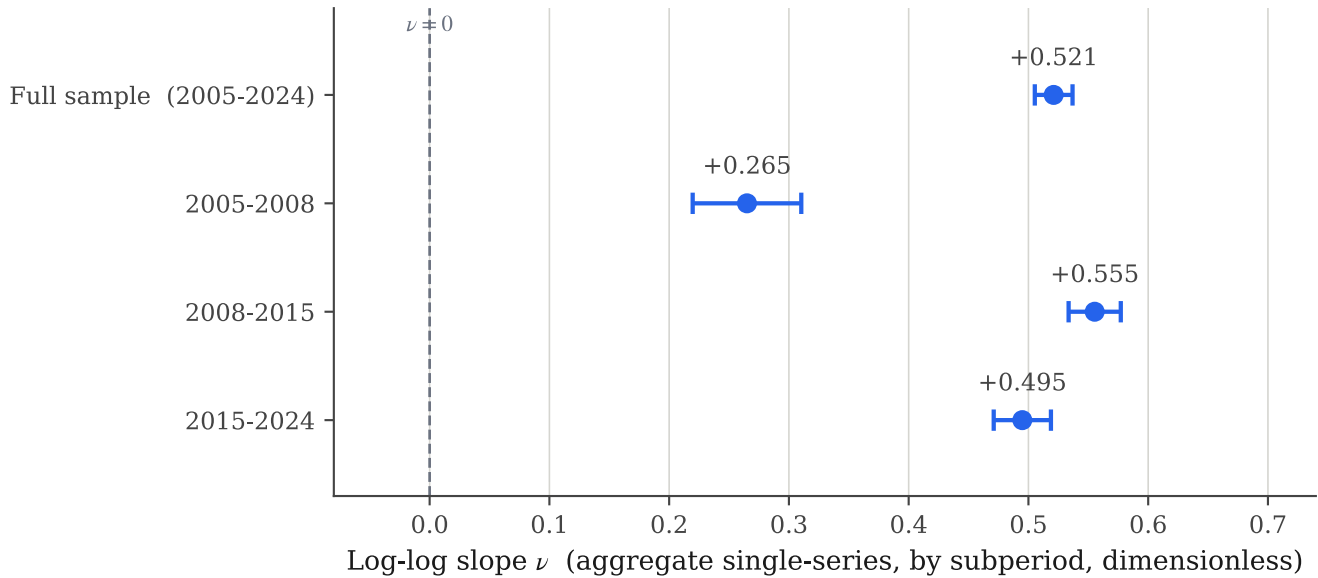


Figure 2. Temporal stability of the aggregate single-series slope ν , estimated separately on the full sample and three consecutive subperiods, with 95% confidence intervals from the stored standard errors. Every interval lies to the right of the dashed sign boundary at $\nu = 0$; the slope is significantly positive in all four windows, though smaller and less precisely estimated in the short 2005–2008 window. Data: the original single-series regression results file (April 2026).

4. Theoretical Reconciliation

4.1 The Goldstone-mode reading (Reading A)

The canonical treatment of Nelson’s osmotic velocity at a symmetry-breaking transition treats it as a Goldstone-mode order-parameter dispersion. In the symmetric phase, the order-parameter modes are gapless and their fluctuations dominate the density gradient; the osmotic velocity scale is large because the density ρ is highly non-uniform across modes. In the broken phase, the Goldstone mode is gapped (the longitudinal direction along the condensate is suppressed by the broken symmetry), and the osmotic velocity scale is small because the density is more uniform. Critical scaling theory predicts

$$|u| \sim (|\Phi|^2)^{-\nu} \quad (\nu > 0 \text{ standard})$$

with $\nu > 0$ the standard critical exponent. This predicts a *negative* log-log slope between $|u|$ and $|\Phi|^2$. The empirics disconfirm this.

4.2 The crowded-trade reading (Reading B)

The alternative reading treats $|\Phi|^2$ as a market-coordination signal rather than a thermodynamic order parameter. When directional consensus is high ($|\Phi|^2$ large), the cross-sectional density ρ concentrates on a few preferred asset configurations — a crowded long/short pattern across the universe. The density-gradient norm $\nabla\rho/\rho$ rises with this concentration, and the osmotic velocity rises with it.

$$|u| \sim (|\Phi|^2)^{+\nu} \quad (\nu > 0 \text{ financial-market form})$$

The empirics confirm this. The slope of approximately +0.5 to +0.6 is consistent with a moderate positive correlation between order-parameter strength and order-flow noise — not a critical scaling exponent in the thermodynamic sense, but an empirical regularity of financial-market microstructure under coordinated positioning.

We record the epistemic order plainly. The study pre-registered the canonical directional prediction $\nu < 0$ (Reading A), with the confirmation criterion a negative slope at $|t| > 3.0$. That hypothesis is disconfirmed in every specification. Reading B is the market-specific interpretation adopted after the sign came out positive; it is consistent with the data but was not itself an out-of-sample prediction, and we therefore do not claim the physics correspondence as established.

4.3 The open question

The two readings differ in their interpretation of the financial “condensate.” In Reading A, $|\Phi|^2$ is a stabilising thermodynamic order parameter for which large values indicate consensus and small values indicate volatility. In Reading B, $|\Phi|^2$ is a destabilising market-coordination indicator for which large values indicate positioning fragility (crowded trades produce large density gradients and large osmotic flows).

Theoretical ambiguity. The financial market is not a closed quantum system in thermal equilibrium. The standard Goldstone-mode prediction comes from equilibrium statistical mechanics. The crowded-trade reading is a non-equilibrium microstructure phenomenon. Resolving which is the correct interpretation requires either: (i) a derivation of the non-equilibrium osmotic-velocity scaling in the financial market context, or (ii) a controlled comparison with financial markets known to be in different microstructure regimes (high-frequency vs low-frequency; constrained-by-shorting vs unconstrained; algorithmic vs discretionary). Both are beyond the empirical scope of this paper.

5. Implications and Discussion

5.1 For the Bohmian-finance research programme

The empirical confirmation of Reading B suggests that financial markets do not behave like equilibrium quantum systems in their osmotic dynamics. This is consistent with the broader non-equilibrium framing of finance under the Valentini H-theorem (Andreev & Howden, 2026f): the market is $\rho \neq |\psi|^2$, and the osmotic velocity is a measurable indicator of how far from equilibrium the system sits at a given moment.

5.2 For market microstructure

The result is also testable against existing literature on crowded trades (Khandani & Lo, 2007; Stein, 2009). In those studies, crowded positioning produces self-reinforcing liquidations during stress regimes. The +0.5–0.6 osmotic–SSB slope quantifies one measurable aspect of this dynamic: when factor alignment grows, the noise floor in cross-sectional positioning grows with it.

5.3 As a market-state indicator

Operationally, the consistent positive slope can be used to construct a regime-state indicator: when realised osmotic velocity outpaces the $|\Phi|^2$ -implied scaling, the market is in an unusually noisy regime relative to its degree of coordination. We note this as a candidate hypothesis for future registration; it carries no registered identifier at the time of writing and is not tested empirically in this paper.

6. Limitations and Future Work

- (1) The five-asset universe is small. Extending to a wider universe (50 or 100 ETFs, or single-name equities) would refine the cluster-robust standard errors and test whether the result is universe-size-dependent.
- (2) The 63-day condensate window and 21-day osmotic window are one of many reasonable choices. We have not run a window-length grid, and we make no claim here that the +0.5 slope is invariant to it; a formal robustness sweep is deferred to a follow-up.

(3) The osmotic-velocity computation uses a density-gradient proxy (local scale times standardised-return magnitude, root-mean-squared across assets); alternative non-parametric estimators (kernel-based; Lyons-signature-based) may yield different slope coefficients. We have not run the comparison.

(4) The theoretical reconciliation between Reading A and Reading B is the central open question. We invite contributions from quantitative-finance theorists familiar with both the condensed-matter Goldstone-mode treatment and financial-market microstructure.

(5) The estimates reported here come from the canonical four-specification pipeline. An earlier single-series computation on a coarser condensate proxy returned a much steeper slope ($v \approx +3.4$); under the canonical pipeline that estimate compresses into the +0.5 range and is superseded. The headline is the +0.50 to +0.61 cross-specification result, not the earlier inflated figure.

7. Discussion

This paper reports an empirical regularity (consistent positive slope across four specifications, $|t| > 12$ in every case) and uses it to discriminate between two theoretical readings of an underlying physical framework. The result confirms that the osmotic velocity rises with the SSB condensate magnitude in financial markets — the opposite direction predicted by the canonical Goldstone-mode reading. The methodological finding is that the two readings are empirically distinguishable, and that the answer is non-trivial: financial markets do not obey the equilibrium-statistical-mechanics scaling prediction.

We deliberately leave the theoretical reconciliation open. The empirical finding is robust; the explanation is not. Honest disclosure of theoretical ambiguity is a feature of methods papers, and we follow that principle here.

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This paper reports the osmotic–SSB scaling relationship across four independent specifications and identifies a theoretical ambiguity between the canonical Goldstone-mode and the crowded-trade readings of the result. The empirics confirm the crowded-trade reading ($v > 0$); the theoretical reconciliation is left open. Correspondence: M.A. Andreev, Quark Quantitative Research.